Math 110

Winter 2021

Lecture 14

Clear all lists,

Store the following in L1

2, 4, 6, 8

Use L1 to find

$\boxed{\mu = 5}$ $\sigma = 2.236$ $\boxed{\sigma^2 (\text{exact}) = 5}$

Take all samples of $\boxed{\text{Size } 2}$ with replacement

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

Now Let's find $\bar{x}$ for each sample

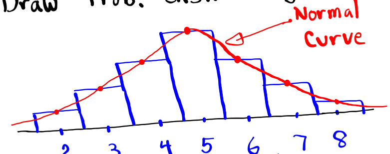
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 $\bar{x}$

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Draw Prob. dist. histogram



Use L2 & L3 to find

$\boxed{\mu = 5}$ $\sigma = 1.581$ $\boxed{\sigma^2 = \frac{5}{2}}$

Clear all lists again

Store 1, 5, 9, 13, and 17 in L1. Use L1 to find

$\boxed{\mu = 9}$ $\sigma = 5.657$ $\boxed{\sigma^2(\text{exact}) = 32}$

Take all samples of $\boxed{\text{Size } 2}$ with replacement from this data

1,1	1,5	1,9	1,13	1,17
5,1	5,5	5,9	5,13	5,17
9,1	9,5	9,9	9,13	9,17
13,1	13,5	13,9	13,13	13,17
17,1	17,5	17,9	17,13	17,17

Let's find $\bar{x}$ of each sample:

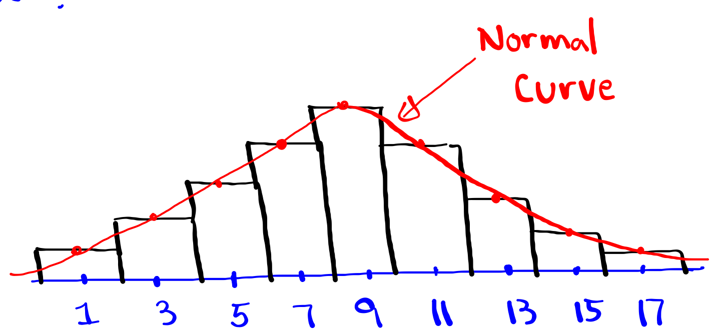
1	3	5	7	9
3	5	7	9	11
5	7	9	11	13
7	9	11	13	15
9	11	13	15	17

25 $\bar{x}$

$\bar{x}$	$P(\bar{x})$
1	1/25
3	2/25
5	3/25
7	4/25
9	5/25
11	4/25
13	3/25
15	2/25
17	1/25

$\bar{x}$	$P(\bar{x})$
1	1/25
3	2/25
5	3/25
7	4/25
9	5/25
11	4/25
13	3/25
15	2/25
17	1/25

Draw Prob. Dist. histogram using $\bar{x}$ & $P(\bar{x})$



$\bar{x} \to L2$ $P(\bar{x}) \to L3$

Use L2 & L3 to find

$\boxed{\mu = 9}$ $\sigma = 4$ $\boxed{\sigma^2 = 16 = \frac{32}{2}}$

Central Limit Theorem:

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Exam 1 results are normally distributed with the mean of 86 and standard deviation 8. $N(86, 8)$

If we randomly select $\boxed{\text{4 exams}}$, find the Prob. that $\underbrace{\boxed{\text{their mean}}}_{\bar{x}}$ score is between 84 and 90.

$P(84 < \bar{x} < 90)$

$= \text{normalcdf}(84, 90, 86, 4)$

$= \boxed{.533}$

$\mu_{\bar{x}} = \mu = 86$

CLT $\left\{ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4 \right.$

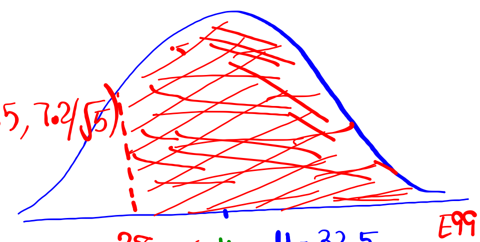
Ages of all Mt.SAC Students are normally distributed with the mean of 32.5 Yrs and standard deviation of 7.2 Yrs. $N(32.5, 7.2)$

If we randomly select $\boxed{\text{5 Students}}$, find the Probability that $\underbrace{\boxed{\text{their mean}}}_{\bar{x}}$ age is above 28.

$P(\bar{x} > 28)$

$= \text{normalcdf}(28, E99, 32.5, 7.2/\sqrt{5})$

$= \boxed{.919}$



$$\text{CLT} \begin{cases} \mu_{\bar{x}} = \mu = 32.5 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{7.2}{\sqrt{5}} \end{cases}$$

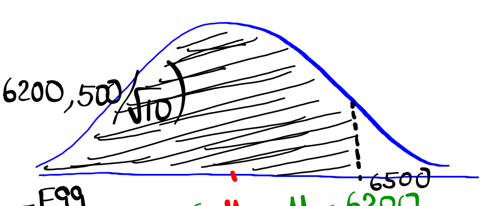
Salaries of nurses are normally distributed with the mean of \$6200 and Standard deviation of \$500. $N(6200, 500)$

If we randomly select $\boxed{\text{10 nurses}}$ find the Prob. that $\underbrace{\boxed{\text{their mean}}}_{\bar{x}}$ Salary is below \$6500.

$P(\bar{x} < 6500)$

$= \text{normalcdf}(-E99, 6500, 6200, 500/\sqrt{10})$

$= \boxed{.971}$



$$\text{CLT} \begin{cases} \mu_{\bar{x}} = \mu = 6200 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{500}{\sqrt{10}} \end{cases}$$

$N(8.75, 4.25)$

A certain brand of dishwashers has a life age that are normally distributed with mean of 8.75 Yrs and standard deviation of 4.25 Yrs.

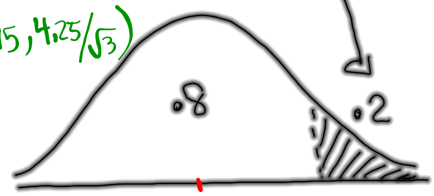
For randomly selected 13 of such dishwashers find the mean age round to 1-decimal that separates the top 20% from the rest.

$\bar{x} = \text{invNorm}(.8, 8.75, 4.25/\sqrt{3})$

Left Area

σ

$= 10.815 \approx \boxed{10.8}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 8.75 \quad \bar{x} \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.25}{\sqrt{3}} \end{cases}$

SAT Scores are normally distributed with the mean of 1125 and standard deviation of 80.

$N(1125, 80)$

If we randomly select 16 SAT Scores find the prob. that their mean score is below 1100 or above 1160.

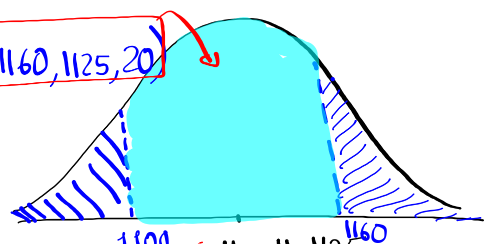
$\bar{x}$

$P(\bar{x} < 1100 \text{ OR } \bar{x} > 1160)$

$= 1 - \boxed{\text{normalcdf}(1100, 1160, 1125, 20)}$

Total Area

$= \boxed{.146}$



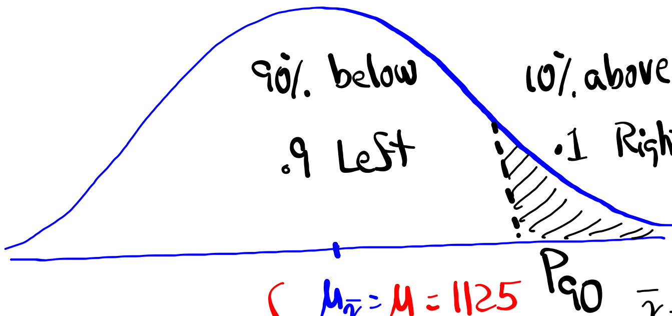
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1125 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{80}{\sqrt{16}} = \frac{80}{4} = 20 \end{cases}$

For randomly selected $\boxed{10 \text{ SAT exams}}$, find the $\bar{x}$ for 90th Percentile.

$\bar{x} = P_{90}$

$P_{90} \to$ 90% below, .9 left

$P_{90} \to$ 10% above, .1 Right



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1125 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{80}{\sqrt{10}} \end{cases}$

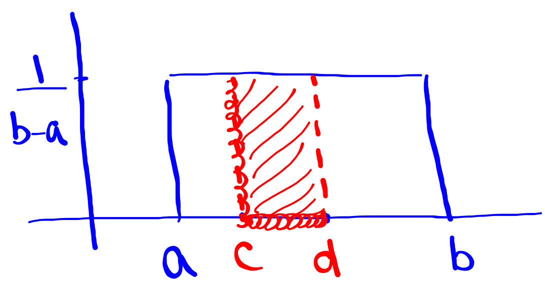
$\bar{x} = \text{invNorm}(.9, 1125, 80/\sqrt{10})$

$\approx \boxed{1157}$

Uniform Prob. dist.

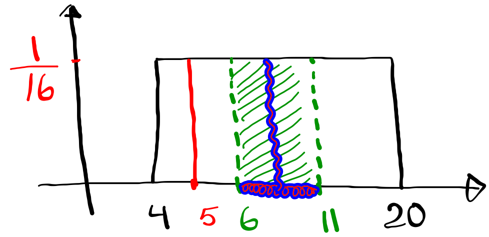
All values from a to b $\Rightarrow a \le x \le b$

Drawing: Rectangle



$P(c < x < d) = L \cdot W$

Consider a uniform Prob. dist for $4 \le x \le 20$

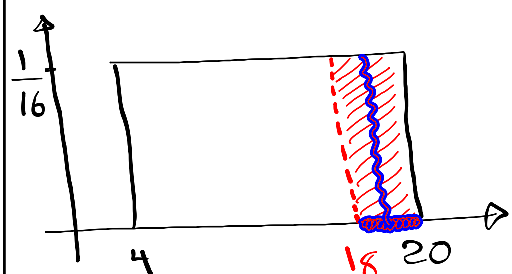


1) $P(x=5)=0$

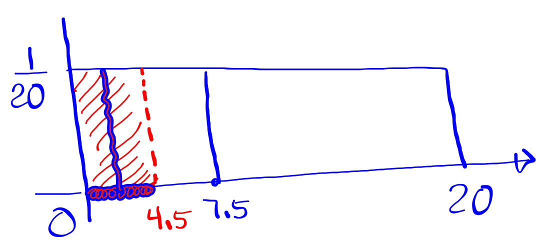
2) $P(6<x<11)$

$= 5 \cdot \frac{1}{16} = \frac{5}{16}$

3) $P(x>18) = (20-18)\cdot\frac{1}{16} = \frac{2}{16} = \boxed{\frac{1}{8}}$



Consider a uniform Prob. dist for $0 \le x \le 20$



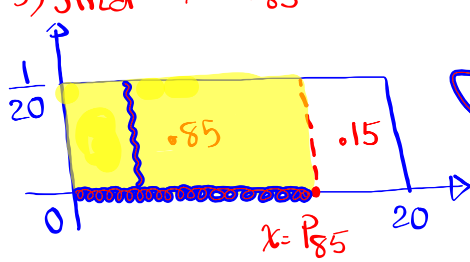
1) $P(x=7.5)=0$

2) $P(x<4.5)$

$=(4.5-0)\cdot\frac{1}{20}$

$=\frac{4.5}{20}=\boxed{.225}$

3) Find $x = P_{85}$



$(x-0)\cdot\frac{1}{20} = .85$

$x \cdot \frac{1}{20} = .85$

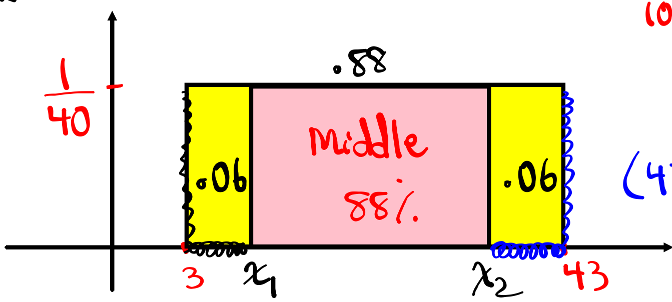
$x = 20(.85)$

$\boxed{x=17}$

Consider a uniform Prob. dist for all values from 3 to 43. Find two values that separate the middle 88% from the rest.

$100\% - 88\% = 12\%$

$12\% \div 2 = 6\%$



$(43 - x_2) \cdot \frac{1}{40} = .06$

$\Rightarrow$ Solve for x_2

$(x_1 - 3) \cdot \frac{1}{40} = .06 \Rightarrow$ Solve for x_1

SG 19
Page 1 & 2.

Bob, the officer, writes 20 speeding tickets in 8-hour shift.

1) Average # of tickets per hour.

$\frac{20 \text{ Tkts}}{8 \text{ hours}} = 2.5 \text{ Tkts/hr} \quad \mu$

2) P(He writes 4 tickets)

$P(x=4) = \text{PoissonPdf}(2.5, 4) = \boxed{.134}$

3) P(He writes fewer than 4 tickets)

$P(x<4) = P(x \leq 3) = \text{PoissonCdf}(2.5, 3)$

$= \boxed{.758}$

4) P(He writes more than 4 tickets)

$P(x>4) = P(x \geq 5) = 1 - P(x \leq 4)$

$= 1 - \text{PoissonCdf}(2.5, 4) = \boxed{.109}$